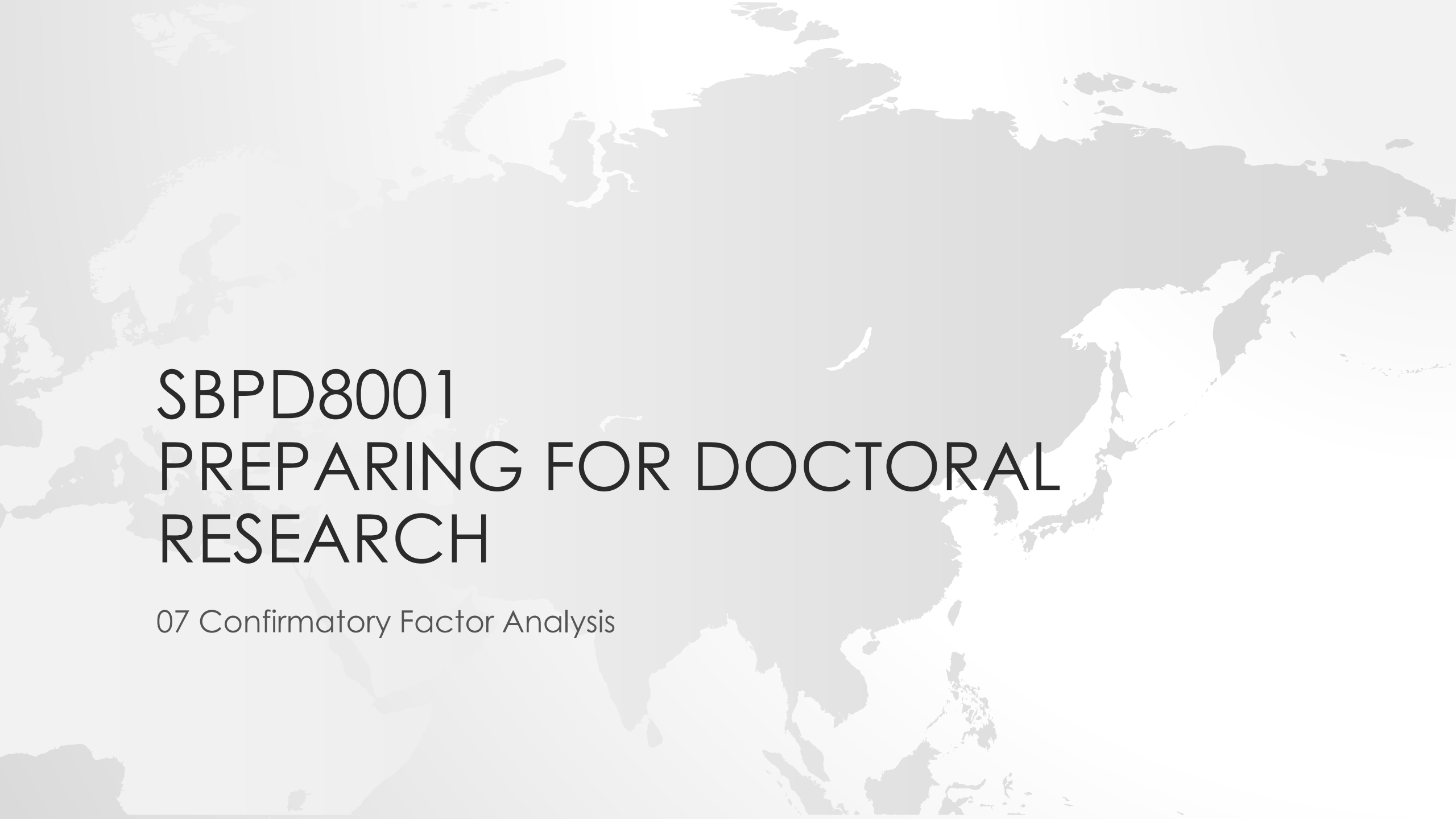


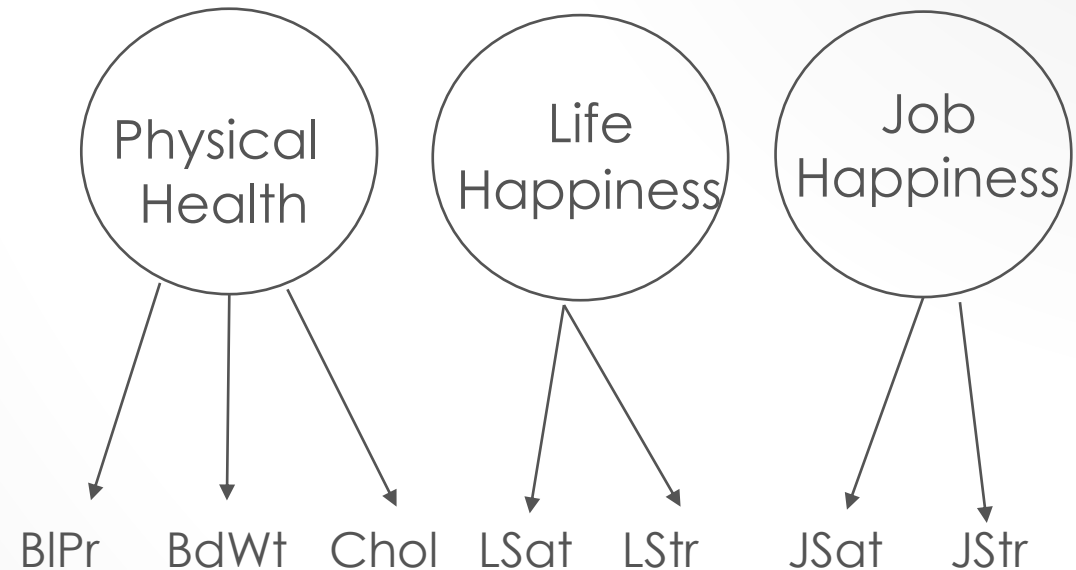
A faint, light gray world map is visible in the background, centered on the Atlantic Ocean. The map shows the outlines of continents and major landmasses.

# SBPD8001 PREPARING FOR DOCTORAL RESEARCH

07 Confirmatory Factor Analysis

# ESSENCE OF CONFIRMATORY FACTOR ANALYSIS

- A structural model is hypothesized in advance
- This model specifies:
  - The number of factors
  - The relationship between observed variables and factors
  - Error terms which comprise unique factors plus measurement error



## Physical Health

BIPr: blood pressure  
Chol: cholesterol  
BdWt: body weight

## Life Happiness

Lsat: Life satisfaction  
LStr: life stress

## Job Happiness

Jsat: Job satisfaction  
JStr: Job stress

# ESSENCE OF CONFIRMATORY FACTOR ANALYSIS

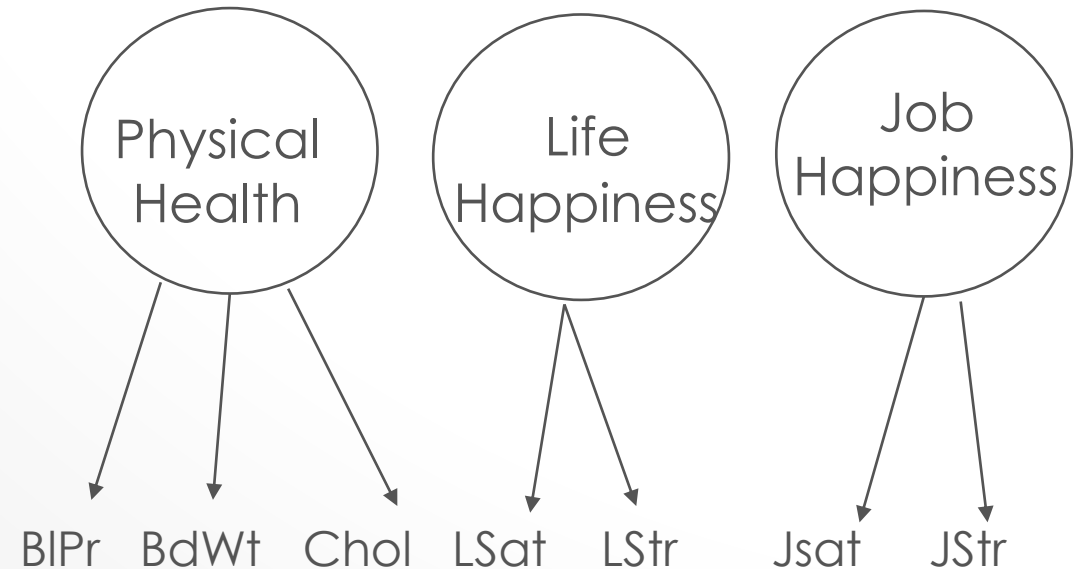
- Part of construct validation process (does the data conform to expectations regarding the underlying patterns?)
- Use SEM packages to perform CFA
- Begin with a correlation matrix and expected relationships
- Look at whether expected relationships can reproduce the correlation matrix well
- Tested with chi-square goodness of fit. If significant ( $p < .05$ ), data don't fit expected structure. No confirmation.
- Alternative measures of fit available.

# ESTIMATION OF A FACTOR MODEL

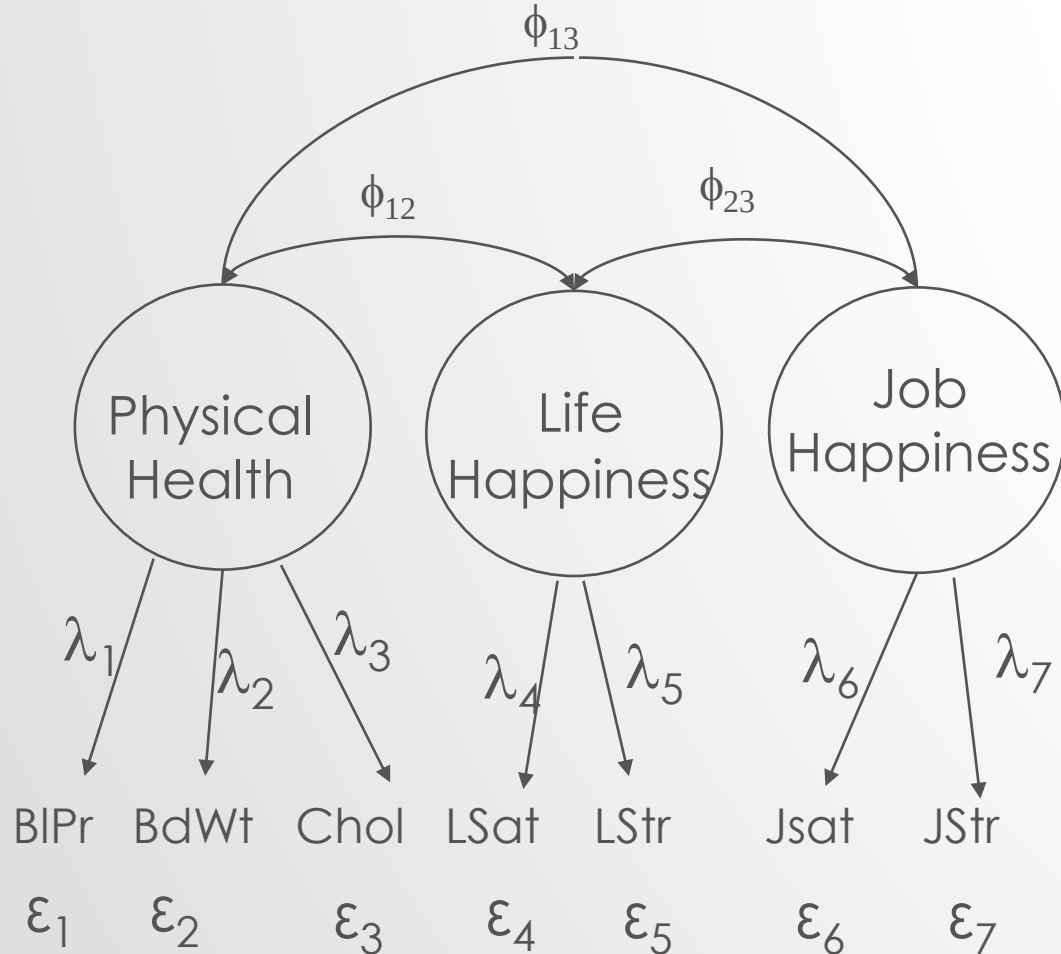
- Data is a sample correlation matrix
- The model is specified a priori:
  - The number of factors
  - The relationship between observed variables and factors
  - Error terms which comprise unique factors plus measurement error
- Model parameters (factor loadings and error variances) are estimated so that the factor model generates a correlation matrix which is as close an approximation to the sample correlation matrix as possible. This is called the model implied correlation matrix.

Sample correlation matrix (1)

|      | BIPr | Lsat | Chol | LStr | BdWt | Jsat | JStr |
|------|------|------|------|------|------|------|------|
| BIPr | 1.00 |      |      |      |      |      |      |
| LSat | -.18 | 1.00 |      |      |      |      |      |
| Chol | .65  | -.17 | 1.00 |      |      |      |      |
| LStr | .15  | -.45 | .22  | 1.00 |      |      |      |
| BdWt | .45  | -.11 | .52  | .16  | 1.00 |      |      |
| JSat | -.21 | .85  | -.12 | -.35 | -.05 | 1.00 |      |
| JStr | .19  | -.21 | .02  | .79  | .19  | .35  | 1.00 |



# ESTIMATION OF A FACTOR MODEL

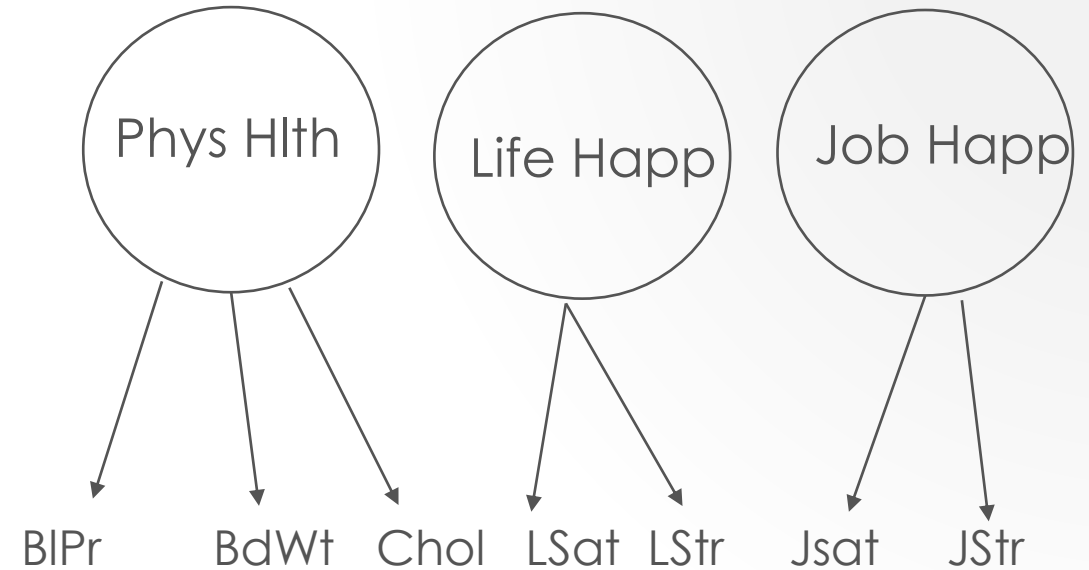


## Model Definition Equations

- $BIPr = \lambda_1 PH + \epsilon_1$
- $BDWt = \lambda_2 PH + \epsilon_2$
- $Chol = \lambda_3 PH + \epsilon_3$
- $LSat = \lambda_4 LH + \epsilon_4$
- $LStr = \lambda_5 LH + \epsilon_5$
- $Jsat = \lambda_6 JH + \epsilon_6$
- $JStr = \lambda_7 JH + \epsilon_7$

### Sample correlation matrix (1)

|      | BIPr | Lsat | Chol | LStr | BdWt | Jsat | JStr |
|------|------|------|------|------|------|------|------|
| BIPr | 1.00 |      |      |      |      |      |      |
| LSat | -.18 | 1.00 |      |      |      |      |      |
| Chol | .65  | -.17 | 1.00 |      |      |      |      |
| LStr | .15  | -.45 | .22  | 1.00 |      |      |      |
| BdWt | .45  | -.11 | .52  | .16  | 1.00 |      |      |
| JSat | -.21 | .85  | -.12 | -.35 | -.05 | 1.00 |      |
| JStr | .19  | -.21 | .02  | .79  | .19  | .35  | 1.00 |



### Implied or fitted correlation matrix (2)

|      | BIPr | Lsat | Chol | LStr | BdWt | Jsat | JStr |
|------|------|------|------|------|------|------|------|
| BIPr | 1.00 |      |      |      |      |      |      |
| LSat | -.17 | 1.03 |      |      |      |      |      |
| Chol | .65  | -.21 | 1.00 |      |      |      |      |
| LStr | .18  | -.45 | .22  | 1.01 |      |      |      |
| BdWt | .45  | -.12 | .53  | .14  | 1.00 |      |      |
| JSat | -.23 | .80  | -.10 | -.36 | -.04 | 1.02 |      |
| JStr | .20  | -.24 | .04  | .77  | .19  | .34  | 1.04 |

### Residuals (3)

|      | BIPr | Lsat | Chol | LStr | BdWt | Jsat | JStr |
|------|------|------|------|------|------|------|------|
| BIPr | .00  |      |      |      |      |      |      |
| LSat | .01  | -.03 |      |      |      |      |      |
| Chol | .00  | .04  | .00  |      |      |      |      |
| LStr | -.03 | .00  | .00  | -.01 |      |      |      |
| BdWt | .00  | .01  | -.01 | .02  | .00  |      |      |
| JSat | .02  | .05  | -.02 | .01  | -.01 | -.02 |      |
| JStr | .01  | .03  | -.02 | .02  | .00  | .01  | -.04 |

**Estimation:** The analysis uses an iterative procedure to minimize the differences between the sample correlation matrix and the estimated population correlation matrix. Discrepancy in fit is represented by the fitted residual matrix.

$$(1) - (2) = (3)$$

The differences between the sample correlation matrix and the implied correlation matrix.

# MODEL FIT

- The goodness-of-fit of confirmatory factor analysis models can be evaluated (not possible for exploratory factor analysis)
- Assess overall model fit with chi-square test
  - $H_0$  = population covariance matrix = covariance matrix based on the parameters defining the model
  - $H_1$  = population covariance matrix  $\neq$  covariance matrix based on the parameters defining the model
  - The null hypothesis is simply that the hypothesized model fits the data
  - If the null hypothesis is not rejected ( $p > .05$ ), an acceptable model fit can be concluded.
- Note: The chi-square test is sensitive to sample size and the number of indicator variables, which means that for large samples even very small differences may be statistically significant.

# FIT STATISTICS

| Measure                       | Name                                         | Description                                                                                                                                                                                          | Cut-off for good fit      |
|-------------------------------|----------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------|
| <b>X<sup>2</sup></b>          | Model Chi-Square                             | Assess overall fit and the discrepancy between the sample and fitted covariance matrices. Sensitive to sample size.<br>H0: The model fits perfectly.                                                 | p-value > 0.05            |
| <b>X<sup>2</sup>/df ratio</b> | Chi-square /degree of freedom ratio          | A ratio of 5 to 1 was "a useful rule of thumb" (Jackson et al., 1993, p. 755).                                                                                                                       | X <sup>2</sup> /df < 5    |
| <b>GFI</b><br><b>AGFI</b>     | Goodness of Fit<br>Adjusted GFI              | GFI is the proportion of variance accounted for by the estimated population covariance. Analogous to R <sup>2</sup> . AGFI favours parsimony.                                                        | GFI ≥ 0.95<br>AGFI ≥ 0.90 |
| <b>(N)NFI</b><br><b>TLI</b>   | (Non) Normed-Fit Index<br>Tucker Lewis index | An NFI of .95, indicates the model of interest improves the fit by 95% relative to the null model. NNFI is preferable for smaller samples. Sometimes the NNFI is called the Tucker Lewis index (TLI) | NFI ≥ 0.95<br>NNFI ≥ 0.95 |

# FIT STATISTICS

| Measure       | Name                                     | Description                                                                                                                                                                                                                         | Cut-off for good fit |
|---------------|------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------|
| <b>CFI</b>    | Comparative Fit Index                    | A revised form of NFI. Not very sensitive to sample size. Compares the fit of a target model to the fit of an independent, or null, model.                                                                                          | $CFI \geq .90$       |
| <b>RMSEA</b>  | Root Mean Square Error of Approximation  | A parsimony-adjusted index. Values closer to 0 represent a good fit.                                                                                                                                                                | $RMSEA < 0.08$       |
| <b>(S)RMR</b> | (Standardized) Root Mean Square Residual | The square-root of the difference between the residuals of the sample covariance matrix and the hypothesized model. If items vary in range (i.e. some items are 1-5, others 1-7) then RMR is hard to interpret, better to use SRMR. | $SRMR < 0.08$        |

# USEFUL RESOURCES

## StatWiki

- [http://statwiki.kolobk creations.com/index.php?title=Confirmator\\_y\\_Factor\\_Analysis](http://statwiki.kolobk creations.com/index.php?title=Confirmator_y_Factor_Analysis)

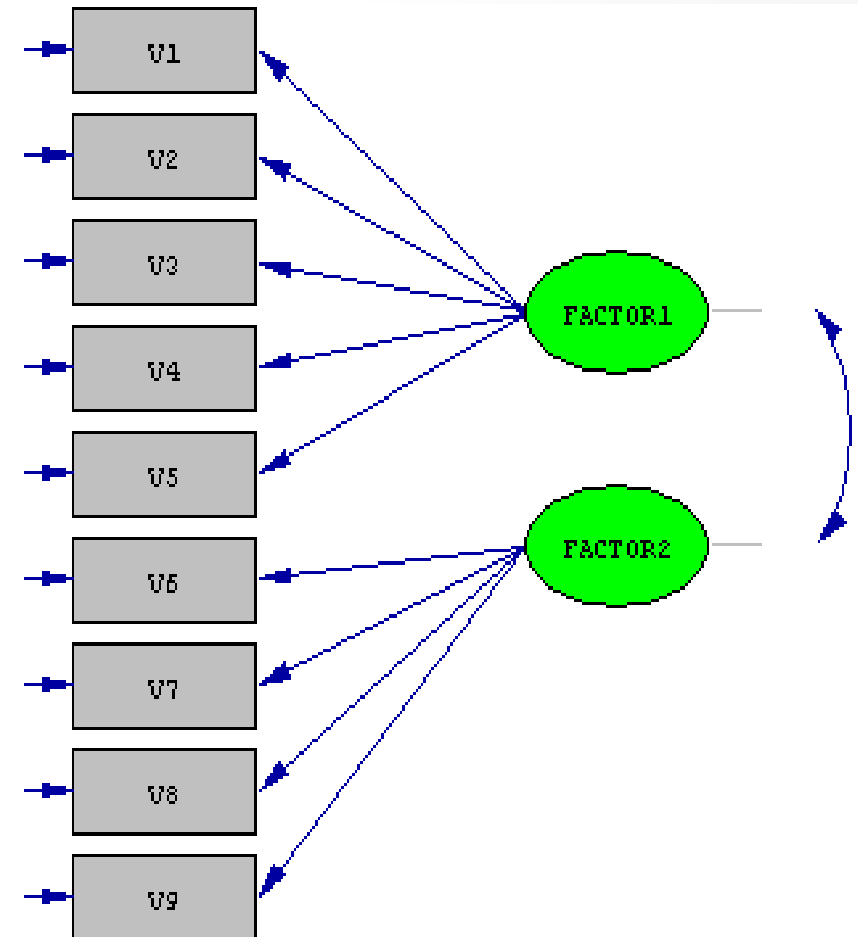
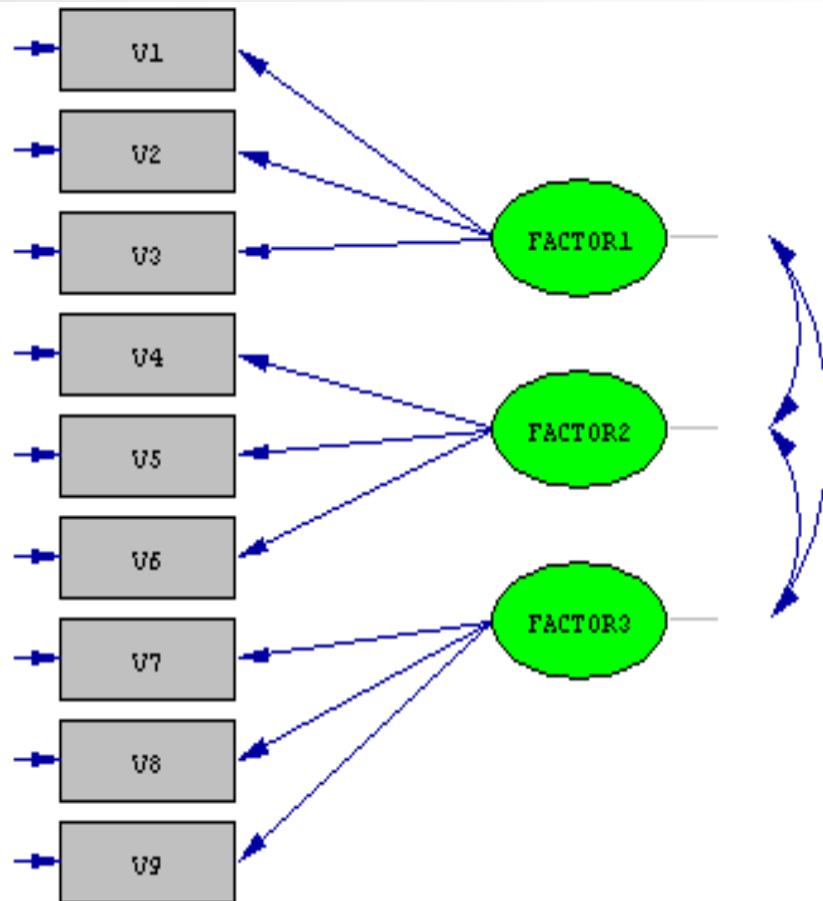
## Model fit during a Confirmatory Factor Analysis (CFA) in AMOS

- <https://www.youtube.com/watch?v=JkZGWUUjdLg>

## How to specify a CFA using AMOS

- <https://www.youtube.com/watch?v=BjvHeRSUuNk>
- Diamantopoulos, A. & Siguaw, J. A. (2013). Introducing LISREL: A guide for the uninitiated. Sage.

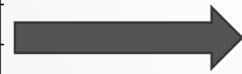
# WHICH MODEL BETTER FITS THE DATA?



# DATA

| v1 | v2 | v3 | v4 | v5 | v6 | v7 | v8 | v9 |
|----|----|----|----|----|----|----|----|----|
| 3  | 3  | 3  | 3  | 5  | 4  | 1  | 1  | 1  |
| 5  | 5  | 6  | 5  | 6  | 4  | 2  | 2  | 2  |
| 3  | 4  | 4  | 6  | 6  | 4  | 1  | 1  | 1  |
| 6  | 6  | 6  | 7  | 6  | 7  | 2  | 4  | 1  |
| 4  | 5  | 6  | 6  | 5  | 5  | 1  | 1  | 1  |
| 4  | 5  | 5  | 5  | 5  | 4  | 5  | 5  | 4  |
| 3  | 4  | 4  | 4  | 5  | 5  | 1  | 1  | 1  |
| 5  | 4  | 5  | 6  | 6  | 5  | 2  | 2  | 2  |
| 5  | 5  | 4  | 4  | 5  | 4  | 1  | 1  | 1  |
| 3  | 4  | 4  | 5  | 5  | 5  | 2  | 2  | 2  |
| 5  | 5  | 6  | 6  | 6  | 5  | 1  | 1  | 1  |
| 4  | 3  | 5  | 5  | 5  | 3  | 1  | 1  | 1  |
| 7  | 7  | 7  | 7  | 5  | 7  | 2  | 2  | 2  |
| 5  | 5  | 5  | 5  | 6  | 6  | 3  | 3  | 3  |
| 5  | 5  | 5  | 6  | 6  | 4  | 4  | 4  | 5  |
| 6  | 6  | 6  | 6  | 6  | 6  | 1  | 1  | 1  |
| 5  | 5  | 5  | 4  | 6  | 4  | 1  | 1  | 1  |
| 5  | 4  | 3  | 5  | 6  | 5  | 3  | 3  | 3  |
| 5  | 6  | 6  | 5  | 6  | 7  | 2  | 2  | 2  |
| 5  | 5  | 5  | 5  | 5  | 4  | 4  | 2  | 4  |

Raw data



|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 1.01           |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | 0.32           | 1.50           |                |                |                |                |                |                |                |
| V <sub>3</sub> | 0.43           | 0.40           | 1.22           |                |                |                |                |                |                |
| V <sub>4</sub> | 0.38           | 0.25           | 0.33           | 1.13           |                |                |                |                |                |
| V <sub>5</sub> | 0.30           | 0.20           | 0.30           | 0.70           | 1.06           |                |                |                |                |
| V <sub>6</sub> | 0.33           | 0.22           | 0.38           | 0.72           | 0.69           | 1.12           |                |                |                |
| V <sub>7</sub> | 0.20           | 0.08           | 0.07           | 0.20           | 0.27           | 0.20           | 1.30           |                |                |
| V <sub>8</sub> | 0.33           | 0.19           | 0.22           | 0.09           | 0.22           | 0.12           | 0.69           | 1.07           |                |
| V <sub>9</sub> | 0.52           | 0.27           | 0.36           | 0.33           | 0.37           | 0.29           | 0.50           | 0.62           | 1.16           |

Sample covariance matrix

Estimation: The analysis uses an iterative procedure to minimize the differences between the sample covariance matrix and the estimated population covariance matrix. Discrepancy in fit is represented by the fitted residual matrix

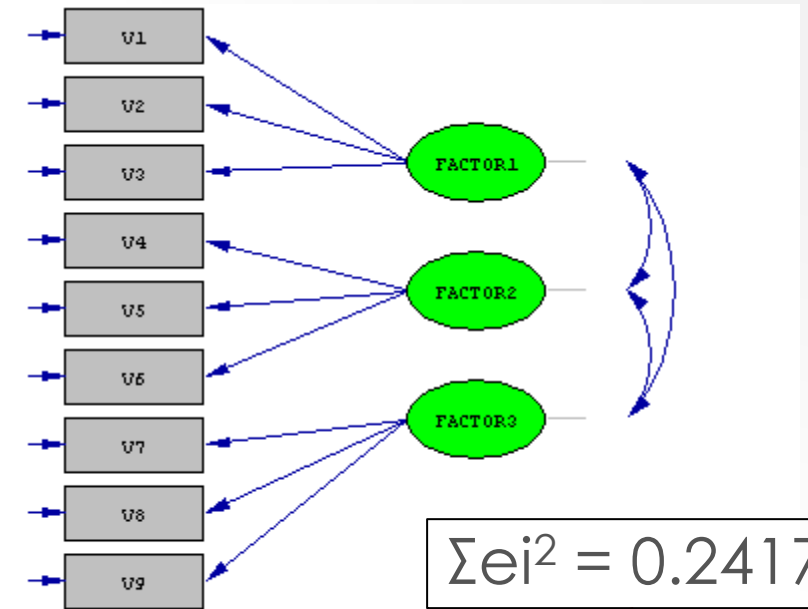
## Three-Factor Model

### Sample covariance matrix

|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 1.01           |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | 0.32           | 1.50           |                |                |                |                |                |                |                |
| V <sub>3</sub> | 0.43           | 0.40           | 1.22           |                |                |                |                |                |                |
| V <sub>4</sub> | 0.38           | 0.25           | 0.33           | 1.13           |                |                |                |                |                |
| V <sub>5</sub> | 0.30           | 0.20           | 0.30           | 0.70           | 1.06           |                |                |                |                |
| V <sub>6</sub> | 0.33           | 0.22           | 0.38           | 0.72           | 0.69           | 1.12           |                |                |                |
| V <sub>7</sub> | 0.20           | 0.08           | 0.07           | 0.20           | 0.27           | 0.20           | 1.30           |                |                |
| V <sub>8</sub> | 0.33           | 0.19           | 0.22           | 0.09           | 0.22           | 0.12           | 0.69           | 1.07           |                |
| V <sub>9</sub> | 0.52           | 0.27           | 0.36           | 0.33           | 0.37           | 0.29           | 0.50           | 0.62           | 1.16           |

### Fitted covariance matrix (3-factor)

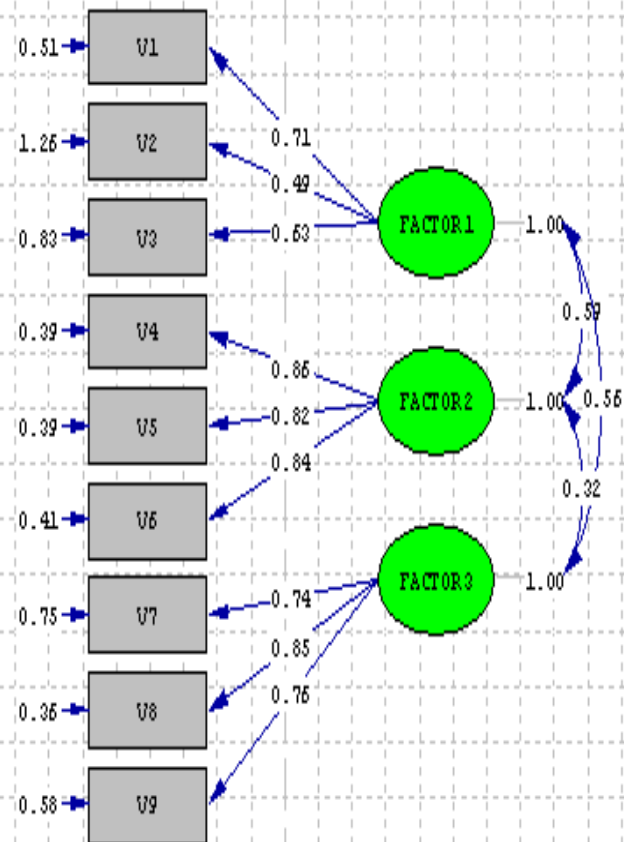
|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 1.01           |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | 0.35           | 1.5            |                |                |                |                |                |                |                |
| V <sub>3</sub> | 0.44           | 0.31           | 1.22           |                |                |                |                |                |                |
| V <sub>4</sub> | 0.36           | 0.25           | 0.32           | 1.13           |                |                |                |                |                |
| V <sub>5</sub> | 0.34           | 0.23           | 0.30           | 0.70           | 1.06           |                |                |                |                |
| V <sub>6</sub> | 0.35           | 0.24           | 0.31           | 0.72           | 0.69           | 1.12           |                |                |                |
| V <sub>7</sub> | 0.29           | 0.20           | 0.26           | 0.20           | 0.19           | 0.20           | 1.3            |                |                |
| V <sub>8</sub> | 0.34           | 0.23           | 0.3            | 0.23           | 0.22           | 0.23           | 0.63           | 1.07           |                |
| V <sub>9</sub> | 0.30           | 0.21           | 0.27           | 0.21           | 0.20           | 0.20           | 0.56           | 0.64           | 1.16           |



### Fitted residual matrix (3-factor)

|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 0              |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | -0.03          | 0              |                |                |                |                |                |                |                |
| V <sub>3</sub> | -0.01          | 0.09           | 0              |                |                |                |                |                |                |
| V <sub>4</sub> | 0.02           | 0              | 0.01           | 0              |                |                |                |                |                |
| V <sub>5</sub> | -0.04          | -0.03          | 0              | 0              | 0              |                |                |                |                |
| V <sub>6</sub> | -0.02          | -0.02          | 0.07           | 0              | 0              | 0              |                |                |                |
| V <sub>7</sub> | -0.09          | -0.12          | -0.19          | 0              | 0.08           | 0              | 0              |                |                |
| V <sub>8</sub> | -0.01          | -0.04          | -0.08          | -0.14          | 0              | -0.11          | 0.06           | 0              |                |
| V <sub>9</sub> | 0.22           | 0.06           | 0.09           | 0.12           | 0.17           | 0.09           | -0.06          | -0.02          | 0              |
|                |                |                |                |                |                | Σ ei =         | 0.07           | Σ ei² =        | 0.2417         |

# THREE-FACTOR MODEL



Chi-Square=66.96, df=24, P-value=0.00001, RMSEA=0.086

1. Degrees of Freedom = 24
2. Minimum Fit Function Chi-Square = 67.71 (P = 0.00)
3. Normal Theory Weighted Least Squares Chi-Square = 66.96 (P = 0.00)
4. Root Mean Square Error of Approximation (RMSEA) = 0.086
5. Chi-Square for Independence Model with 36 Degrees of Freedom = 972.12
6. Root Mean Square Residual (RMR) = 0.074
7. Goodness of Fit Index (GFI) = 0.94

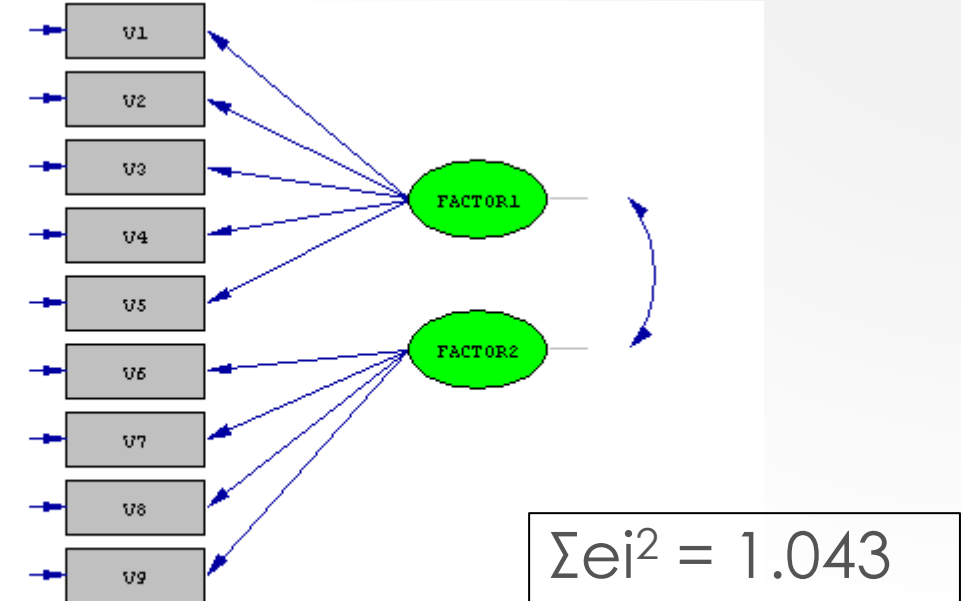
## Sample covariance matrix

|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 1.01           |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | 0.32           | 1.50           |                |                |                |                |                |                |                |
| V <sub>3</sub> | 0.43           | 0.40           | 1.22           |                |                |                |                |                |                |
| V <sub>4</sub> | 0.38           | 0.25           | 0.33           | 1.13           |                |                |                |                |                |
| V <sub>5</sub> | 0.30           | 0.20           | 0.30           | 0.70           | 1.06           |                |                |                |                |
| V <sub>6</sub> | 0.33           | 0.22           | 0.38           | 0.72           | 0.69           | 1.12           |                |                |                |
| V <sub>7</sub> | 0.20           | 0.08           | 0.07           | 0.20           | 0.27           | 0.20           | 1.30           |                |                |
| V <sub>8</sub> | 0.33           | 0.19           | 0.22           | 0.09           | 0.22           | 0.12           | 0.69           | 1.07           |                |
| V <sub>9</sub> | <b>0.52</b>    | <b>0.27</b>    | <b>0.36</b>    | <b>0.33</b>    | <b>0.37</b>    | <b>0.29</b>    | <b>0.50</b>    | <b>0.62</b>    | <b>1.16</b>    |

## Fitted covariance matrix (2-factor)

|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 1.01           |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | 0.17           | 1.50           |                |                |                |                |                |                |                |
| V <sub>3</sub> | 0.24           | 0.16           | 1.22           |                |                |                |                |                |                |
| V <sub>4</sub> | 0.40           | 0.28           | 0.38           | 1.13           |                |                |                |                |                |
| V <sub>5</sub> | 0.40           | 0.27           | 0.38           | 0.64           | 1.06           |                |                |                |                |
| V <sub>6</sub> | 0.41           | 0.28           | 0.39           | 0.67           | 0.66           | 1.12           |                |                |                |
| V <sub>7</sub> | 0.17           | 0.12           | 0.16           | 0.28           | 0.27           | 0.26           | 1.30           |                |                |
| V <sub>8</sub> | 0.16           | 0.11           | 0.15           | 0.25           | 0.25           | 0.23           | 0.10           | 1.07           |                |
| V <sub>9</sub> | <b>0.27</b>    | <b>0.18</b>    | <b>0.25</b>    | <b>0.43</b>    | <b>0.42</b>    | <b>0.40</b>    | <b>0.17</b>    | <b>0.15</b>    | <b>1.16</b>    |

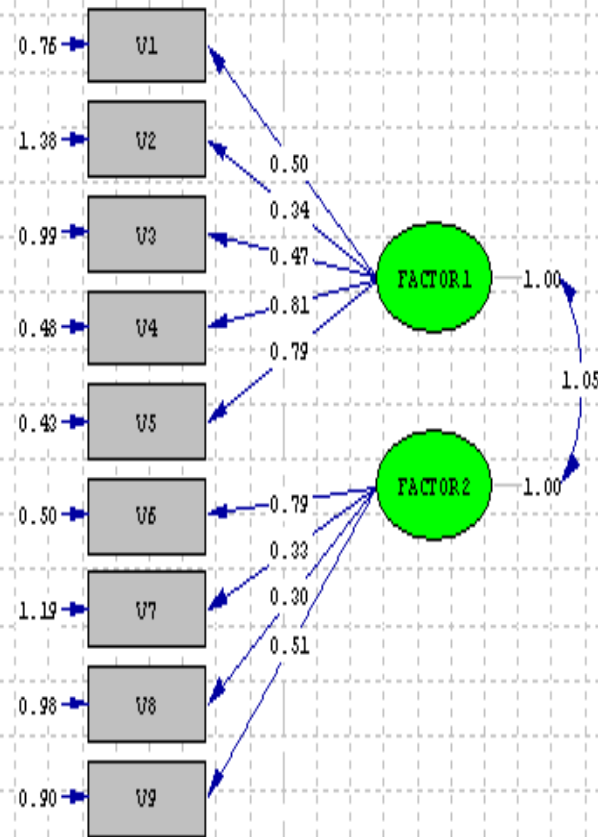
## Two-Factor Model



## Fitted residual matrix (2-factor)

|                | V <sub>1</sub> | V <sub>2</sub> | V <sub>3</sub> | V <sub>4</sub> | V <sub>5</sub> | V <sub>6</sub> | V <sub>7</sub> | V <sub>8</sub> | V <sub>9</sub> |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| V <sub>1</sub> | 0              |                |                |                |                |                |                |                |                |
| V <sub>2</sub> | 0.15           | 0              |                |                |                |                |                |                |                |
| V <sub>3</sub> | 0.19           | 0.24           | 0              |                |                |                |                |                |                |
| V <sub>4</sub> | -0.02          | -0.03          | -0.05          | 0              |                |                |                |                |                |
| V <sub>5</sub> | -0.1           | -0.07          | -0.08          | 0.06           | 0              |                |                |                |                |
| V <sub>6</sub> | -0.08          | -0.06          | -0.01          | 0.05           | 0.03           | 0              |                |                |                |
| V <sub>7</sub> | 0.03           | -0.04          | -0.09          | -0.08          | 0              | -0.06          | 0              |                |                |
| V <sub>8</sub> | 0.17           | 0.08           | 0.07           | -0.16          | -0.03          | -0.11          | 0.59           | 0              |                |
| V <sub>9</sub> | 0.25           | 0.09           | 0.11           | -0.1           | -0.05          | -0.11          | 0.33           | 0.47           | 0              |
|                |                |                |                |                |                | Σ ei =         | 1.58           | Σ ei² =        | 1.043          |

# TWO-FACTOR MODEL



Chi-Square=298.66, df=26, P-value=0.00000, RMSEA=0.207

1. Degrees of Freedom = 26
2. Minimum Fit Function Chi-Square = 261.74 (P = 0.0)
3. Normal Theory Weighted Least Squares Chi-Square = 298.66 (P = 0.0)
4. Root Mean Square Error of Approximation (RMSEA) = 0.21
5. Chi-Square for Independence Model with 36 Degrees of Freedom = 972.12
6. Root Mean Square Residual (RMR) = 0.15
7. Goodness of Fit Index (GFI) = 0.79

# FIT STATISTICS

|                                                              | Three-factor model | Two-factor model  | Cut-off for good fit |
|--------------------------------------------------------------|--------------------|-------------------|----------------------|
| Degrees of Freedom                                           | 24                 | 26                |                      |
| Minimum Fit Function Chi-Square                              | 67.71 (P = 0.00)   | 261.741 (P = 0.0) | p > 0.5              |
| X <sup>2</sup> /df ratio                                     | 2.82               | 10.06             | < 5                  |
| Normal Theory Weighted Least Squares Chi-Square              | 66.96 (P = 0.00)   | 298.66 (P = 0.0)  |                      |
| Root Mean Square Error of Approximation (RMSEA)              | 0.086              | 0.21              | < 0.08               |
| Chi-Square for Independence Model with 36 Degrees of Freedom | 972.12             | 972.12            |                      |
| Root Mean Square Residual (RMR)                              | 0.074              | 0.15              | < 0.08               |
| Goodness of Fit Index (GFI)                                  | 0.94               | 0.79              | > 0.95               |

# PRESENTATION OF RESULTS

- The three-factor model produced better fitting indices ( $\chi^2=67.71$ ,  $df=24$ ,  $p<.00$ , Standardized RMR=.074, RMSEA=.086, GFI=.94). Although the chi-square statistic was significant, all other model fit indices provided evidence of good model fit.

|       | $V_1$                           | $V_2$                           | $V_3$                           | $V_4$                           | $V_5$                           | $V_6$                           | $V_7$                    |
|-------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|--------------------------|
| $V_1$ | $\lambda_1^2 + \theta_1$        |                                 |                                 |                                 |                                 |                                 |                          |
| $V_2$ | $\lambda_1 \lambda_2$           | $\lambda_2^2 \theta_2$          |                                 |                                 |                                 |                                 |                          |
| $V_3$ | $\lambda_1 \lambda_3$           | $\lambda_2 \lambda_3$           | $\lambda_3^2 + \theta_3$        |                                 |                                 |                                 |                          |
| $V_4$ | $\lambda_1 \lambda_4 \phi_{21}$ | $\lambda_2 \lambda_4 \phi_{21}$ | $\lambda_3 \lambda_4 \phi_{21}$ | $\lambda_4^2 + \theta_4$        |                                 |                                 |                          |
| $V_5$ | $\lambda_1 \lambda_5 \phi_{21}$ | $\lambda_2 \lambda_5 \phi_{21}$ | $\lambda_3 \lambda_5 \phi_{21}$ | $\lambda_4 \lambda_5$           | $\lambda_5^2 + \theta_5$        |                                 |                          |
| $V_6$ | $\lambda_1 \lambda_6 \phi_{21}$ | $\lambda_2 \lambda_6 \phi_{21}$ | $\lambda_3 \lambda_6 \phi_{21}$ | $\lambda_4 \lambda_6$           | $\lambda_5 \lambda_6$           | $\lambda_6^2 + \theta_6$        |                          |
| $V_7$ | $\lambda_1 \lambda_7 \phi_{31}$ | $\lambda_2 \lambda_7 \phi_{31}$ | $\lambda_3 \lambda_7 \phi_{31}$ | $\lambda_4 \lambda_7 \phi_{32}$ | $\lambda_5 \lambda_7 \phi_{32}$ | $\lambda_6 \lambda_7 \phi_{32}$ | $\lambda_7^2 + \theta_7$ |

$\theta_1$ =variance of  $\varepsilon_1$

$\phi_{21}$ = covariance of factor1 and factor2